

Effect Of Varying Step Sizes On The Performance Of LMS Based Adaptive Filter Algorithms

Debasmita Basu¹, Writi Mitra², Subhojit Malik³

¹ B.Tech. Student, Electronics and Communication Engineering Department, Hooghly Engineering and Technology College, Hooghly, West Bengal, India

² Assistant Professor, Electronics and Communication Engineering Department, Hooghly Engineering and Technology College, Hooghly, West Bengal, India

³ Assistant Professor, Electronics and Communication Engineering Department, Hooghly Engineering and Technology College, Hooghly, West Bengal, India

Abstract: Any adaptive system is responsible for determining a discrete estimation of transfer function for any unknown analog or digital system. When the parameters of any system are not known, the adaptive filter algorithms are iterative in nature and can be implemented to estimate the unknown parameters. The Adaptive Filtering and the implementation of Adaptive Filtering Algorithm have been discussed in this paper. There are a number of filtering algorithms based on the method of Least Mean Squares and they are: Least Mean Square (LMS), Leaky Least Mean Square (LLMS) and Normalized Least Mean Square (NLMS) Algorithms. In these adaptive algorithms the step size is an important parameter which is varied for a fixed value of number of iterations and number of taps so that it controls the error functions. A comparative study has been made and it is observed how the nature of frequency response curve of each algorithm is changing with the variation of the parameters.

Keywords – Adaptive Filter Algorithm, Least Mean Square (LMS) Algorithm, Leaky Least Mean Square (LLMS) Algorithm, Normalized Least Mean Square (NLMS) Algorithm Frequency Response Curve, Step Size.

I. INTRODUCTION

Digital Signal Processing (DSP) is the mathematical manipulation of an information signal to modify or improve the characteristics of the signals[8]. Any system which is designed based on DSP, can be used to measure, filter, and compress continuous real-world analog signals. The contributions are significantly large in the field of DSP for the last thirty years in the field of signal processing. The design of digital signal processing systems is popular for low cost, reliability, accuracy, small physical sizes and flexibility. Filtering is a signal processing operation whose objective is to process a signal in order to manipulate the information contained in the signal. A filter is used to map the input signal to another output signal facilitating the extraction of the desired information contained in the input signal. The parameters of time-invariant system and the structure of the filters are fixed. The output signal for a system is a linear function of the input signal for a linear system and the output signal is a non-linear function of the input signal for a non-linear system.

An adaptive filter is a filter that self-adjusts its transfer function according to an optimization algorithm driven by an error signal[1,2,8,9,10]. Generally, the adaptive process involves the use of a cost function. To find out the optimum performance of the filter and to feed an algorithm, a cost function is used and it is minimized. It helps to determine and modify filter transfer function to minimize the cost on the next iteration. An adaptive filter is required when either the fixed specifications are unknown or the specifications cannot be satisfied by time-invariant filters. Although the input signals satisfy the homogeneity property but the additive conditions are not satisfied. For this reason the adaptive filter is non-linear. At the same time the adaptive filters are time-varying since their parameters are continually changing in order to meet a performance requirement. An adaptive filter performs the approximation. The definition of the performance criterion requires the existence of a reference signal that is usually hidden in the approximation step for any filter design. This paper deals with the basics of Adaptive Filters, the implementation of Adaptive Filter Algorithms and a comparative analysis of them. There are a number of adaptive filtering algorithms based on the method of Least

Mean Squares, namely, Least Mean Square (LMS), Normalized Least Mean Square (NLMS) and Leaky Least Mean Square (LLMS) Algorithms.

In the next section of the paper, the Adaptive Filtering Algorithms (II), The Implementation of Adaptive Filtering Algorithms (III), The Result Analysis (IV), The Conclusion (V) and The References are given.

II. ADAPTIVE FILTERING ALGORITHMS

The basic adaptive filter algorithm scheme is given below. In this algorithm the number of iteration is taken as n, the input signal is denoted by x(n), the adaptive-filter output signal is y(n) and d(n) is the desired signal. The error signal is e(n) which is calculated as[8-10]

$$e[n] = d[n] - y[n] \text{-----(1)}$$

The error signal is then used to form a performance or objective function which is used by the adaptation algorithm in order to determine the appropriate filter coefficients. The objective function is minimized which implies that the adaptive-filter output signal is matching the desired signal in some sense.

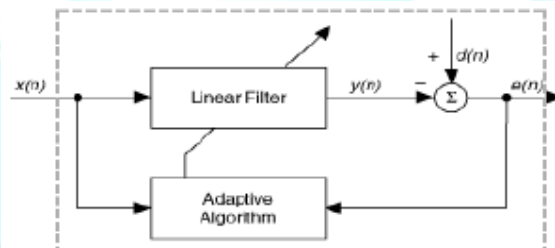


Figure1: Basic Adaptive Filter Algorithm Scheme

The error signal plays a crucial role for the algorithm, since it can affect several characteristics of the overall algorithm including robustness, speed of convergence, computational complexity, and most importantly for the IIR adaptive filtering case, the occurrence of biased and multiple solutions.

In times of minimizing the objective function, the error signal is interpreted, analyzed and studied in the adaptive algorithm. In fact, almost all known adaptive algorithms can be visualized in this form or with slight variation. It may be observed that the minimization of the objective function affect the convergence speed of the adaptive algorithm. It is important to choice the error signal in an adaptive algorithm, since the overall convergence process influence many aspects of the adaptive algorithm.

There are many methods for the performing weight update of an adaptive filter. The different filtering algorithms which are used: Least Mean Square (LMS), Normalized Least Mean Square (NLMS) and Leaky Least Mean Square (LLMS) Algorithms.

LEAST MEAN SQUARE ALGORITHM

Least Mean Square (LMS) was developed by Windrow & Holf in 1959. The LMS algorithm is an approximation of the steepest descent algorithm which uses an instantaneous estimate of the gradient vector of a cost function[2,8-10]. The gradient is estimated based on sample values of the tap-input vector and an error signal. In this algorithm the iterations are taken over each coefficient in the filter and they are moving in the direction of the approximated gradient. For the LMS algorithm it is necessary to have a reference signal d[n] and this reference signal represents the desired filter output. The difference between the reference signal and the actual output is taken as the error signal which is represented in Equation 2.

$$e[n] = d[n] - c^H [n]x[n] \text{-----(2)}$$

In the LMS algorithm, a set of filter coefficients which is represented by c is used and this set of coefficients is minimized for the expected value of the quadratic error signal to achieve the least mean squared error. This is the reason for using the name.

To update the coefficients in LMS algorithm, Equation 3 is used for calculating the updated coefficients at every time instant n,

$$c[n+1] = c[n] + \mu e^*[n]x[n] \text{-----} (3)$$

In this context, the selection of the parameter ‘step-size’ which is denoted by μ , is introduced in Equation 3. The step size controls the movement of coefficients along the error function surface at each updated step. Now the value of μ has to be chosen ($\mu > 0$) in such a way that the coefficient vector will move in a direction towards larger squared error. At the same time, the step size (μ) should not be too large. Since in the LMS algorithm a local approximation is done in the computation of the gradient of the cost function, and thus the cost function at each time instant may differ from the global cost function. Furthermore, a large step-size causes the LMS algorithm to be unstable and the coefficients do not converge to fixed values but it may oscillate.

LEAKY LEAST MEAN SQUARE ALGORITHM

The cost function of the leaky LMS (LLMS) algorithm is defined by the following equation:

$$j[n] = e^2[n] + \gamma \sum_{i=0}^{N-1} w_i^2[n] \text{-----} (4)$$

where γ is the leaky factor and the range of γ is 0 to 0.1. Now another term μ may be present. The cost function of the leaky LMS algorithm is different from the standard LMS algorithm. In leaky LMS algorithm, the coefficients may migrate and it may cause a overflow problem. The cost function of this algorithm accounts for both $e^2(n)$ and the filter coefficients. The leaky LMS algorithm updates the coefficients of an adaptive filter by using the following equation:

$$w[n+1] = (1 - \mu\gamma)w[n] + \mu e[n]u[n] \text{-----} (5)$$

If $\gamma = 0$, the leaky LMS algorithm becomes the same as the standard LMS algorithm. If a large leaky factor is chosen, it may cause a steady state error.

NORMALIZED LEAST MEAN SQUARE ALGORITHM

In case of pure LMS algorithm, there is a disadvantage. The algorithm is sensitive to the scaling of its input $x(n)$. Due to this reason, choosing of a learning rate μ is difficult and the stability of the algorithm may hamper.

The recursion formula for the Normalized Least Mean Square (NLMS) algorithm is stated in equation 6 and it is denoted by

$$w[n+1] = w[n] + \mu[n]e[n]u[n] \text{-----} (6)$$

Here, $\mu[n]$ is the step size and it is varying with the time.

$$\mu[n] = \frac{\beta}{x[n]x^T[n] + c} \text{-----} (7)$$

$\mu[n]$ is denoted by Equation 7 and c is a small positive constant to avoid division by zero and β is normalized step-size nearly varying form $0 < \beta < 2$.

III. IMPLEMENTATION OF ADAPTIVE FILTERING ALGORITHMS

MATLAB (version 7.8) is used for implementing all these adaptive filter algorithms[5,7]. As a test case, FIR low pass filter is used as the desired system to implement the adaptive filter algorithms. To implement this FIR low pass filter, window technique can be used. In this system, Blackman window is taken for test purpose. In FIR filter the impulse response is of finite duration. FIR system is used because it is easy to implement and this filter is inherently stable. Blackman window has been used to get the desired filter coefficient as fixed coefficient. The Blackman window coefficients are expressed as:

$$w[n] = 0.42 - 0.5 \cos\left(\frac{2\pi n}{N-1}\right) + 0.08 \cos\left(\frac{4\pi n}{N-1}\right) \text{-----(8)}$$

where , $0 \leq n \leq N-1$

The input is taken as

$$x[n] = \sin(\omega_c(n - \alpha + \epsilon)) / (\pi * (n - \alpha + \epsilon)) \text{--- (9)}$$

where ω_c is the cut-off frequency, $\epsilon = \text{phase} = 0.001$ and $\alpha = (\text{No. of taps}-1) / 2$;

Finally the LMS, LLMS and NLMS algorithms are implemented by using MATLAB and the system is tested by using the above input by using Blackman window function. A comparative study of system outputs and system parameters are carried out by varying different depending parameters of all the algorithms.

IV. RESULT ANALYSIS

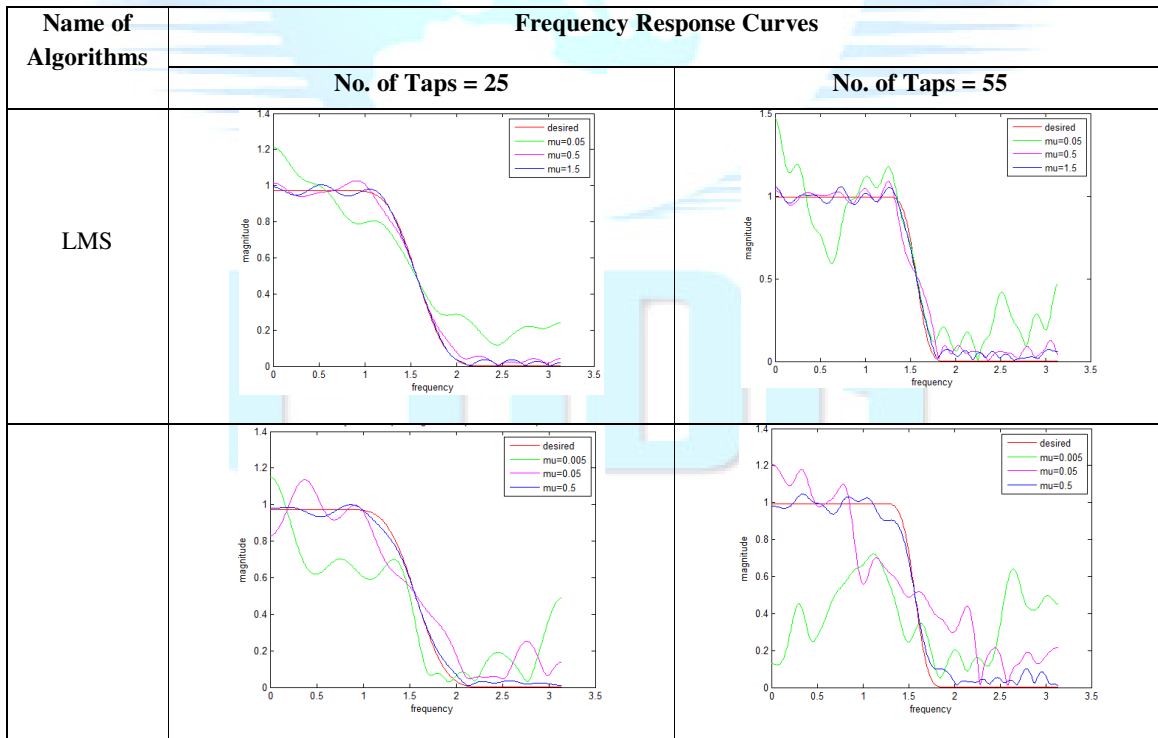
As a test system, a digital low pass filter is used with a Blackman window function. LMS, LLMS and NLMS Algorithms are applied to the system and the frequency response curves for these systems are plotted.

The number of iterations is taken as 1000. The results are checked by varying the number of taps.

For LMS algorithm the results are plotted by varying the step size μ .

For LLMS algorithm, at first the results are plotted by varying the step size μ and second time the results are plotted by varying the value of γ .

In NLMS algorithm, the time varying step size μ consist of β and c . At first the results are plotted by varying the value β and second time the results are plotted by varying the value of c .



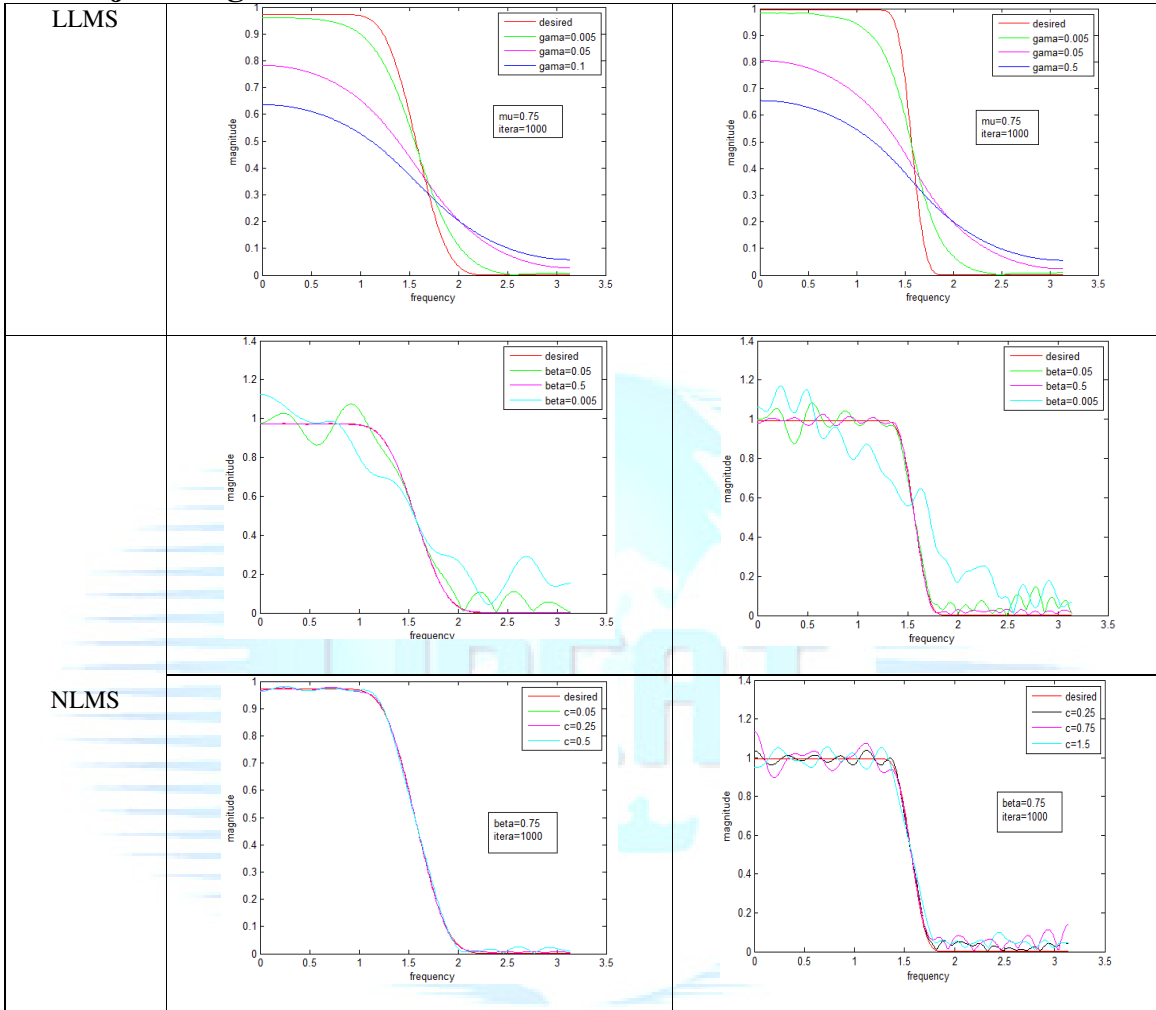


Figure 2: Graph analysis for comparative study with varying step-size with fixed number of steps and iterations

In LMS algorithm, the value of step size is varied for $\mu = 0.05, 0.5, \text{ and } 1.5$. From the plot for 25 taps it is observed that the best response is obtained when the value of step size is 1.5 as compared to its values of others. Similarly in the plot for 55 taps, the best response for step size 1.5 is obtained.

In LLMS algorithm, the value of step size (μ) is varied for $\mu = 0.005, 0.05 \text{ and } 0.5$. From the plot for 25 taps it is observed that the best response is obtained when the value of μ is 0.5 as compared to its values of 0.005 & 0.05. Similarly in the plot for 55 tap, the best response for $\mu = 0.5$ is obtained.

At the same time, the value of γ is varied for 0.005, 0.05 & 0.5. From the plot for tap 25 we can see that we get the best response when the value of γ is 0.005 as compared to its values of 0.05 & 0.5. Similarly in the plot for 55 taps we can see the best response for $\gamma = 0.005$.

In NLMS algorithm, the value of step size (β) is varied for 0.005, 0.05 & 0.5. From the plot, for 25 taps, we get the best response when β is 0.5 as compared to its values of others. Similarly in the plot for 55 taps, we get the best response for $\beta = 0.5$.

At the same time the normalized step size ($\beta = 0.75$) and number of iteration (1000) is fixed for both the plot considering different number of taps as 25 and 55 respectively and the value of c (small positive value) is varied for 0.25, 0.75, and 1.5. We get the best response when c is 0.25 as compared to its values of others.

Similarly, in the plot for 55 taps, we get the best response for c is 0.75.

Now the variations of coefficients of the adaptive filter algorithms with respect to the number of taps are plotted. The number of iterations is taken as 1000 in each of the cases.

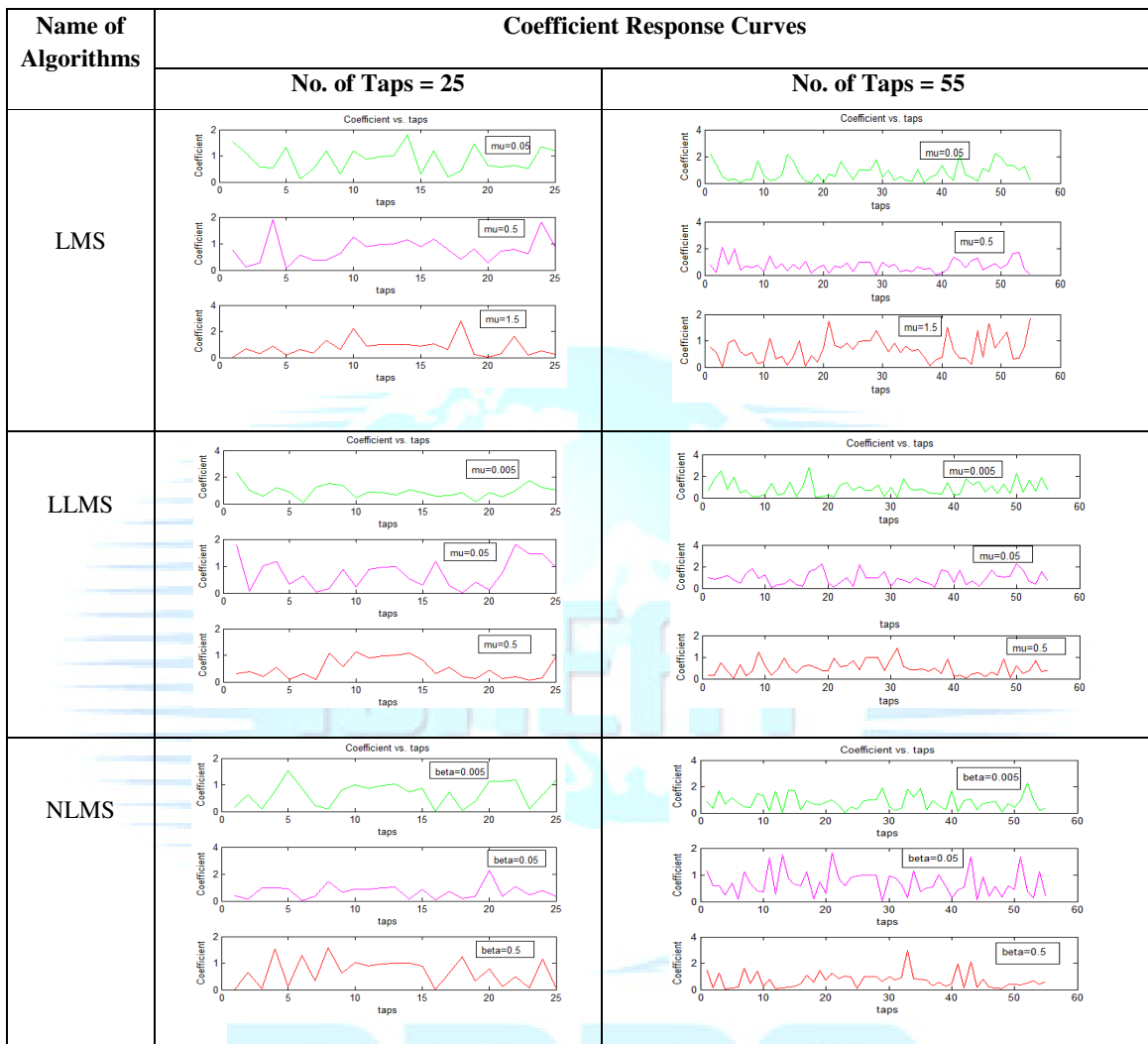


Figure 3: Graphical analysis of coefficient response curves of different adaptive filter algorithms for different step size w.r.t. number of taps

In the Figure 3, for LMS algorithm, the number of iterations is taken as 1000 for both the plots when the number of taps are taken as 25 and 55. The step size is varied for 0.05, 0.5, and 1.5. It is observed that when the number of tap is taken as 25, a better response is obtained when step size is 1.5. As the tap size increase, for fixed iteration, standard LMS response for coefficient becomes poor.

In LLMS algorithm, we can see a response curve for coefficient vs. number of taps. The number of iterations is fixed (1000) for both the plots when the number of iterations are taken as 25 and 55 respectively. In the plots, the value of step size is varied for 0.005, 0.05, and 0.5. As the tap size increase, for fixed iteration, LLMS response becomes poor.

In NLMS algorithm, we have plotted the coefficient vs. number of taps considering the number of iterations is fixed (1000) for both the plots of 25 taps and 55 taps. In plots, value of step size is varied for 0.005, 0.05, and 0.5. As the tap size increase, for fixed iteration, NLMS response becomes poor.

Error is an important factor for any algorithms. Now we consider the amount of error that is introduced in times of implementing the algorithms. The error response curves are now plotted.

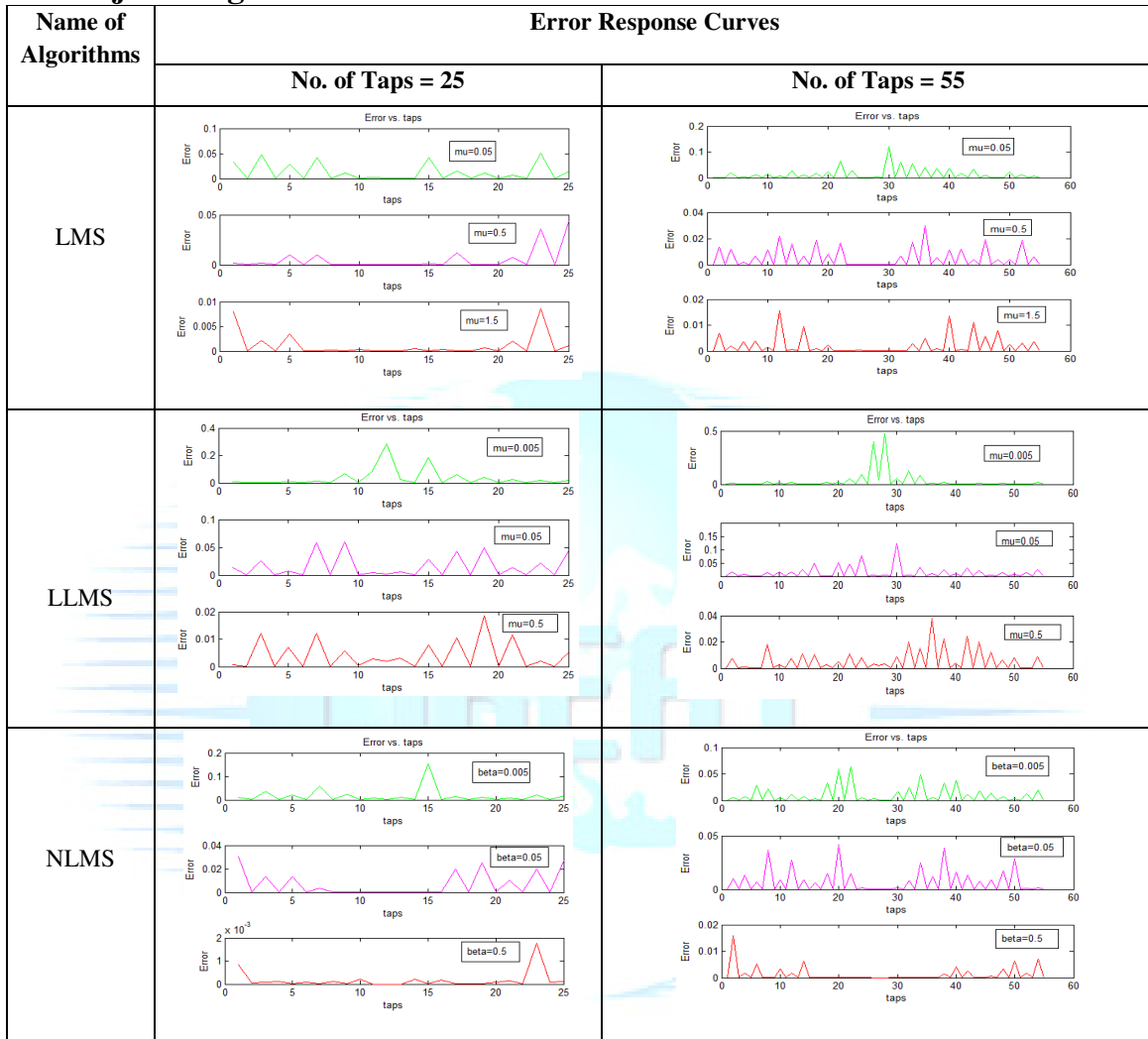


Figure 4: Graphical analysis of different error response curves of adaptive filter algorithms for step size w.r.t. number of taps

In Figure 4, the plot of Error vs. number of taps is done for a fixed number of iterations which is taken as 1000 for all the plots for LMS algorithm. The number of taps is taken as 25 and 55 respectively. In the plots, the step size is varied for 0.05, 0.5, and 1.5. For, tap value 25, we get better response for error because the amount of error is less when step size is 1.5. If the tap size is increased, the amount of error is more for standard LMS response hence the performance becomes poor. So in LMS algorithm, when tap size is increased the efficiency becomes decrease.

In LLMS algorithm, the iteration is again chosen as 1000 for both the cases when the number of taps is 25 and 55 respectively. In the plots, the step-size is varied for 0.005, 0.05 and 0.5. When the step size is less i.e. when step size is considered as 0.005, the amount of error is less. So decreasing step size gives better response for both the cases when the number of taps is 25 and 55 respectively.

In NLMS algorithm, the plot is taken with error vs. number of taps where the number of iterations is taken as 1000 for all the plots and again the number of taps is taken as 25 and 55. In plots, value of step size is varied for 0.005, 0.05, and 0.5. For, tap value 25, we get better response for error curve with less error, when step size is 0.5. As the tap size increase, tap 55 for fixed iteration, NLMS response for error curve with more error, becomes poor.

V. CONCLUSION

In case of LMS based algorithms, it can be concluded that when number of iterations is kept constant and step size is varied for different number of taps 25 and 55 respectively, the best response is obtained when value of

μ is 1.5. Now if both the plots are compared for taps 25 and 55, it is observed that a better response is achieved for $\mu = 1.5$ for taps=25. Thus we can say that we can get the best response when the value of number of taps (N) is 25, value of step-size $\mu = 1.5$. So $0.5 < \mu < 2$ is better for LMS.

When the coefficient response is considered a better response is obtained when step size is 1.5. If the tap size is increased, for fixed iteration, the standard LMS response for coefficient becomes poor for any value of the step size.

In case of error response, for, tap value 25, we get better response for error because the amount of error is less with step size 1.5. If the tap size is increased, the amount of error is more for standard LMS response hence the performance becomes poor. So in LMS algorithm, when tap size is increased the efficiency becomes decrease.

In case of LLMS algorithm, when number of iterations and value of gamma is kept constant and step size (μ) is varied for different number of taps 25 and 55, the best response is obtained when value of μ is 0.5 for taps=25. Now if we compare both the plots for taps 25 and 55 and consider the response for $\mu = 0.5$ in both the plots we can see that we get a better response when the value of number of taps (N) is 25 and step-size $\mu=0.5$. Now if we consider the value of γ for both the plots for number of taps 25 and 55 and consider the response for $\gamma = 0.005$ in both the plots we can see that we get a better response when $\gamma = 0.005$ when number of taps is 25.

In coefficient response curves, if the tap size is increased, for fixed number of iteration, LLMS response becomes poor irrespective of any value of step size.

In case of error response, decreasing step size gives better response for any number of taps.

In case of NLMS algorithm, when number of iterations is kept constant and step size (β) is varied for different number of taps 25 and 55 we get the best response when β is 0.5. Now if we compare both the plots for number of taps 25 and 55 and consider the response for $\beta = 0.5$ in both the plots we can see that we get a better response when $\beta = 0.5$ when number of for taps is 25. Now if we consider the value of c for both the plots for number of taps 25 and 55, it is observed that all the responses are very much close to the desired response when the number of taps is 25.

When the coefficient response is considered, if the tap size is increased, for fixed iteration, NLMS response becomes poor for any value of step size.

In case of error response, we get better response with less error, when step size is 0.5. As the tap size is increased, for fixed iteration, NLMS response for error curve with more error becomes poor.

ACKNOWLEDGEMENTS

The authors would like to thank the Department of Electronics and Communication Engineering and the authority of Hooghly Engineering and Technology College, Hooghly, West Bengal, India for providing the facilities, support and continuous encouragement to continue research work.

REFERENCES

- [1] S.Haykin, Adaptive Filter Theory, Prentice-Hall, 2002.
- [2] Paulo S.R. Diniz, Adaptive Filtering – Algorithms & Practical Implementation, Second Edition, Kluwer Academic Publishers.
- [3] “Performance Comparison of Adaptive Filter Algorithms for ECG Signal Enhancement”, Syed Ateequr Rehman, R.Ranjith Kumar, International Journal of Advanced Research in Computer and Communication Engineering Vol. 1, Issue 2, April 2012.
- [4] Ahmed Elhossini, Shawki Areibi, Robert Dony, “An FPGA Implementation of the LMS Adaptive Filter for Audio Processing”, IEEE International Conference on Reconfigurable Computing and FPGA's, ReConFig 2006
- [5] “VLSI Design and Implementation for Adaptive Filter using LMS Algorithm”, Asit Kumar Subudhi, Biswajit Mishra, Mihir Narayan Mohanty, International Journal of Computer & Communication Technology (IJCCT), Volume-2, Issue-VI, 2011.

- [6] “Modified Adaptive Filtering Algorithm for Noise Cancellation in Speech Signals”, V. R. Vijaykumar, P. T. Vanathi, P. Kanagasabathy, Electronics and Electrical Engineering. – Kaunas: Technologija, 2007. – No. 2(74). – P. 17–20.
- [7] “FPGA Implementation of Adaptive Filter and its Performance Analysis”, J. Malarmanan, S. Malarvizhi, International Journal of Engineering and Technology, Vol 5 No 3 Jun-Jul 2013, ISSN : 0975-4024.
- [8] Dr. Shaila D. Apte, Digital Signal Processing, 2nd Edition, Wiley Publication , 2014
- [9] John G. Proakis, Dimitris G. Manolakis, Digital Signal Processing Principles, Algorithms and Applications, Fourth Edition, PHI Publication, 2006.
- [10] S. Salivahanan, A. Vallavaraj, C. Gnanaapriya, Digital Signal Processing, Tata McGraw-Hill, 2000.

